Inverse Simulation

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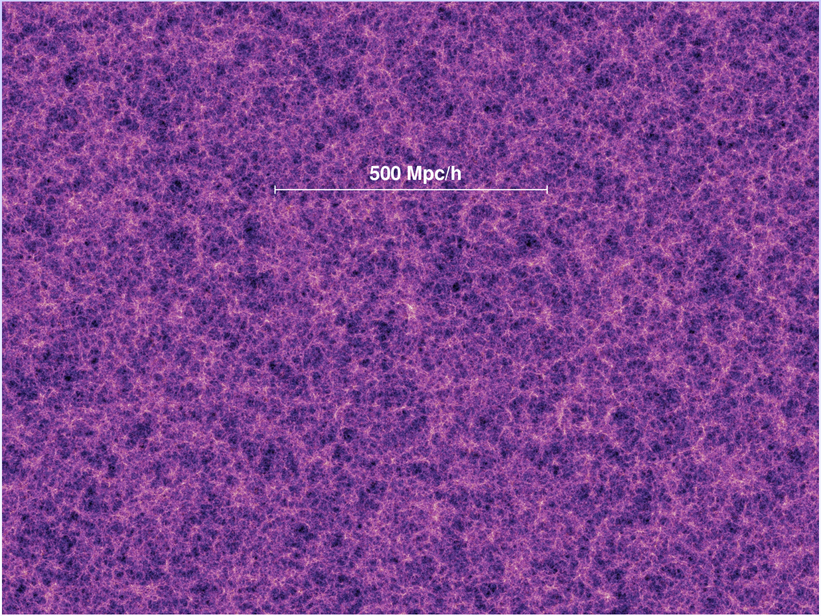
Recap

- N-body simulations are our only tool to understand deeply non-linear structure formation
- In the very early universe, evolution of perturbations can be calculated exactly
- After CMB in the matter domination, linear evolution is very easy: each mode grows independently:

$$\begin{aligned}
\rho(\mathbf{x}) &= \bar{\rho}(1+\delta(\mathbf{x})) && (1)\\
\delta(\mathbf{x}, z) &= G(z)\delta(\mathbf{x}) && (2)\\
\delta_k(\mathbf{k}, z) &= G(z)\delta_k(\mathbf{k}) && (3)\\
P(z, k) &= G(z)^2 \langle \delta_k \delta_k^* \rangle && (4)
\end{aligned}$$

Running simulations

- You form initial conditions by picking a sufficiently high redhift and:
  - Loop over k-modes
  - For each k- mode pick a Gaussian random complex number with variance $P(k)$ (and random phase)
- This gives you a *realization* of the field at given redshift
- Move particles appropriately and give them appropriate linear velocities
- Start a full integration of the system:
  - Large scale remain linear and grow according to $G(z)$
  - Small scales evolve into fully non-linear structure


500 Mpc/h

Phase rotation

We can transform any Gaussian initial conditions of an N-body simulation into an equally likely initial conditions by transforming like

$$\delta_1(\mathbf{k}) \rightarrow \ {}_A\delta_1 = \delta_1 A(\mathbf{k}), \tag{5}$$

If

$$AA^* = 1, \tag{6}$$

$$A(\mathbf{k}) = A^*(-\mathbf{k}). \tag{7}$$

the rotated initial conditions are an equally likely realization of the same universe.

Two special cases:

- $A = \pm 1$, a constant
- $A = e^{i\mathbf{kr}}$, a translation

Inverse simulations

A trivial case $A = -1$. We have

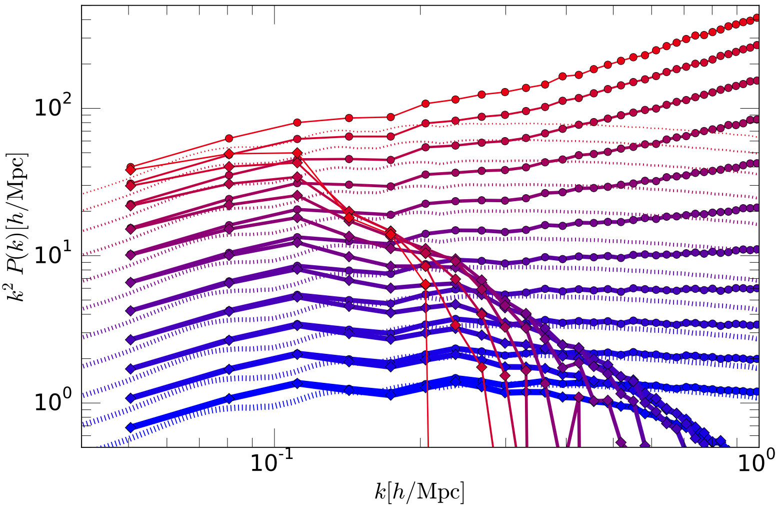
$$\delta \to -\delta \tag{8}$$

(both in real and Fourier space)
You get a pair of initial conditions, but

- Overdensities in correspond to underdensities in the other (and vice versa)
- Halos in one will correspond to voids in the other (and vice versa)
- Large scales are expected to evolve the same (up to a minus sign).
- Small scales will "decorrelate"

We run a pair

- w Pontzen (UCL)
- 200 Mpc, 512^3 particles, WMAP5 cosmology
- Run standard and inverse



$k^2\ P(k)[h/\mathrm{Mpc}]$
10^2
10^1
10^0
10^{-1}
10^0
$k[h/\mathrm{Mpc}]$

Perturbation theory

There are many perturbation theory approaches. In SPT

$$\delta(\mathbf{k}) = \sum_{i=1}^{\infty} a^i \delta_i(\mathbf{k}), \tag{9}$$

where $a\delta_n$ term is a convolution of n initial fields δ_1 with a relevant perturbation theory kernel and where translational invariance reduces the dimensionality of the integral to $n-1$:

$$\delta_n(\mathbf{k}) = \iiint F_n(\mathbf{k}, \mathbf{k}', \mathbf{k}'' \ldots) d^3\mathbf{k}' d^3\mathbf{k}'' d^3\mathbf{k}''' \ldots d^3\mathbf{k}'^{(n-1)\mathrm{times}} \delta_1(\mathbf{k}') \delta(\mathbf{k}'') \tag{10}$$

It is immediately clear that for the inverse simulations, the orders in the evolved field are the same in magnitude, but that odd ones flip the sign:

$$\delta_{\mathrm{i},j} = (-1)^j \delta_j \tag{11}$$

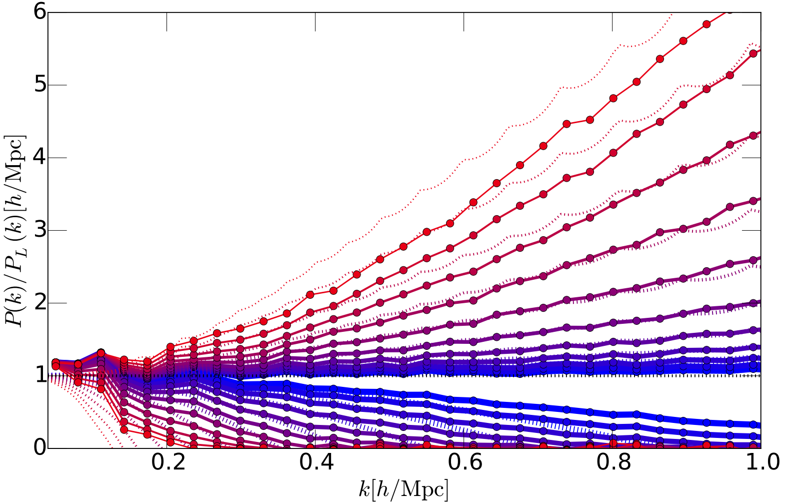
Perturbation theory

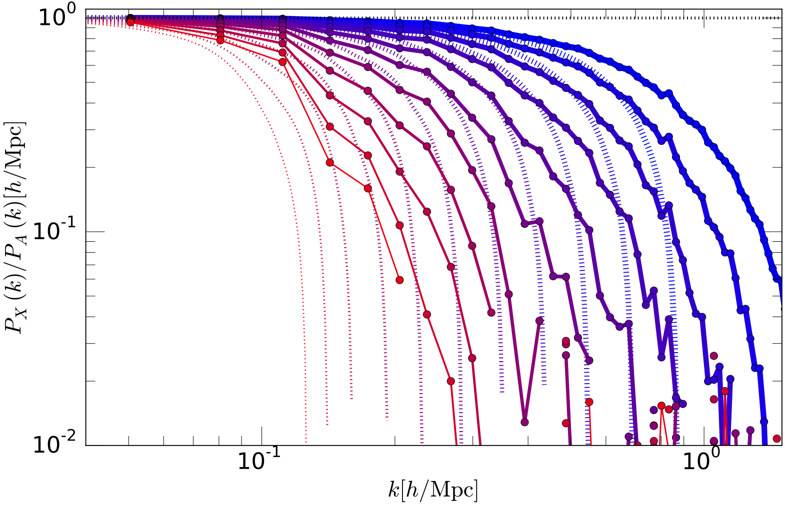
The standard auto-power spectrum is given by

$$P(k) = \langle \delta(\mathbf{k})\delta^*(\mathbf{k}) \rangle = P_{11}(k) + (P_{13}(k) + P_{22}(k)) + \ldots \quad (12)$$

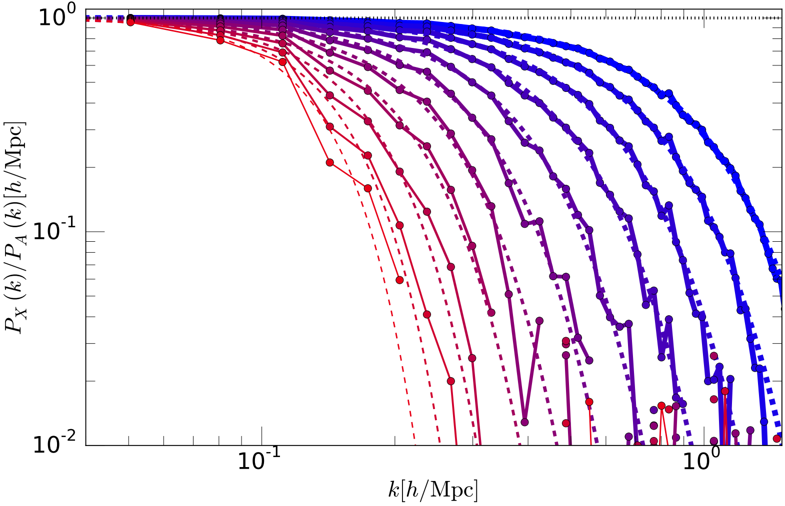
where P_{11} is the linear power spectrum and at second order we get two contributions $P_{22} = \langle \delta_2(k)\delta_2^*(k) \rangle$ and $P_{13} = 2 \langle \delta_1(k)\delta_3^*(k) \rangle$. The cross-power spectrum between the standard and inverse field is the same, but the some terms acquire negative sign:

$$P_X(k) = \langle \delta(\mathbf{k})\delta^*(\mathbf{k}) \rangle = -P_{11}(k) + (-P_{13}(k) + P_{22}(k)) + \ldots \quad (13)$$


6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1.0
$P(k)/P_L(k)[h/\mathrm{Mpc}]$
$k[h/\mathrm{Mpc}]$



10^0
10^{-1}
10^{-2}
$P_X(k)/P_A(k)[h/\mathrm{Mpc}]$
10^{-1}
10^0
$k[h/\mathrm{Mpc}]$



10^0
10^{-1}
10^{-2}
10^{-1}
10^0
$P_X(k)/P_A(k)[h/\mathrm{Mpc}]$
$k[h/\mathrm{Mpc}]$

Fitting supression

We fitted $P_x(k)/P_I(k)$ with the following function fomr:

$$\frac{P_x(k)}{P_A(k)} = e^{-\left(\frac{k}{k_{\mathrm{NL}}}\right)^{\alpha}} \tag{14}$$

We see that this simple formula is an excellent fit down to 6th snapshot corresponding to redshift of $z \sim 2$. In fact, the fitted α is equal to 2 to 4 significant figures at $z = 9$, is around 2.2 at redshift of $z = 2$ and raises to ~ 3 at $z = 0$.
Cleary a deeper reason for Gaussian suppression. Argument why expansion parameter should be small (k/k_{NL}) rather than δ.

Conclusions

- Currently running simulations where just $k < k_\star$ modes are flipped: can test the spreading of information in modes
- Excellent tool for void studies – identify halos in inverse, look where those particle IDs end up in standard
- Very good tool for testing various perturbation schemes, e.g. Effective Field Theory